

All-Optical Control of Light Group Velocity with a Cavity Optomechanical System

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Abstract

We theoretically demonstrate complete all-optical control of light group velocity via a cavity optomechanical system composed of an optical cavity and a mechanical resonator. The realization depends on no specific materials inside the cavity, and the control of light group velocity stems from the interaction between the signal light and the moving optical diffraction grating within the cavity in analogy to the stimulated Brillouin scattering(SBS). Furthermore, we show that a tunable switch from slow light to fast light can be achieved only by simply adjusting the pump-cavity detuning. The scheme proposed here will open a novel way to control light velocity by all-optical methods in optomechanical systems.

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I. INTRODUCTION

The ability to control the velocity of light has attracted a lot of attention from both technical and scientific communities in physics [1]. Devices based on slow and fast light can be used as buffering and regeneration in optical telecommunication, continuously tunable phase shifter in microwave photonics and spectrometer in interferometry [2–5]. Rapid progress of slow and fast light has been made in a variety of media by several physical mechanisms, such as electromagnetically induced transparency (EIT), coherent population oscillation (CPO) and stimulated Brillouin scattering (SBS) [6–10]. Recently, light control in solids has been observed in crystals, erbium doped fibers and photo-refractive materials [11–13]. Moreover, extreme values of group velocity at room temperature, which are suitable for many practical applications, have also been reported [14]. On the other hand, bestriding the realms of classical and quantum mechanics, optomechanical system has been subject to increasing investigation[15, 16]. By coupling a driven high-frequency mode to a high-Q, low-frequency mechanical mode and with the analysis of the intrinsic properties, such system offers great promise for a huge variety of applications and fundamental researches[17–21].

In the present article, we propose a novel all-optical scheme to control light group velocity by using a cavity optomechanical system. Theoretical analysis shows that when the cavity-pump detuning is fixed properly, the cavity optomechanical system, composed of an optical cavity and a mechanical resonator, displays a strong dispersive property with little absorption of signal light, and thus the control of group velocity can be realized. The physical picture behind is, to some extent, similar to the simulated Brillouin scattering which occurs readily in any transparent material. However, the process that the signal light experiences in the system is a complete all-optical process, since no specific material is needed to be placed in the cavity. And, unlike the EIT process, fast light can also be generated in the optomechanical system. Moreover, we find that the scattering of signal light depends on the pump-cavity detuning, hence the switch between slow and fast light can be achieved by adjusting pump-cavity detuning properly.

II. THEORY

We consider the canonical situation in which a driven high-finesse cavity is coupled by momentum transfer of the cavity photons to a micromechanical resonator. The physical realization as shown in Fig.1(a) is a Fabry-Perot cavity being formed on one end by a moving mirror. The cavity free spectrum range $L/2c$ (c is the speed of light in the vacuum and L is the effective cavity length [22, 23]) is much larger than the frequency of movable mirror (ω_m). Therefore the scattering of photons to other cavity modes can be ignored, and we can adopt the single-cavity-mode description. Before we discuss the system's ability to control the group velocity, we start our description with the familiar Hamiltonian in the rotating frame at a pump field frequency ω_{pu} [24, 25]

$$H = \hbar\Delta_{pu}a^\dagger a + \frac{1}{2}\hbar\omega_m(p^2 + q^2) - \hbar G_0 a^\dagger a q + i\hbar E_{pu}(a^\dagger - a) + i\hbar E_s(a^\dagger e^{-i\delta t} - a e^{i\delta t}), \quad (1)$$

where $\Delta_{pu} = \omega_c - \omega_{pu}$ is the detuning of the pump field frequency and the frequency ω_c of the cavity mode with bosonic operators a and a^\dagger . The quadratures q and p associate to the mechanical mode satisfying the usual commutation relations of canonical coordinates. The parameter $G_0 = \frac{\omega_c}{L} \sqrt{\frac{\hbar}{m\omega_m}}$ is the coupling rate between the cavity and the resonator (m is the effective mass of mechanical mode [23]). E_{pu} and E_s are related to the laser power P by $|E_{pu}| = \sqrt{\frac{2P_{pu}\kappa}{\hbar\omega_{pu}}}$ and $|E_s| = \sqrt{\frac{2P_s\kappa}{\hbar\omega_s}}$ respectively (κ is the cavity amplitude decay rate). $\delta = \omega_s - \omega_{pu}$ is the detuning of signal and pump field, ω_s is the signal field frequency.

According to the Heisenberg equation of motion $i\hbar \frac{dO}{dt} = [O, H]$, the temporal evolutions of the lowering operator a and the dimensionless position operator q are given by

$$\frac{da}{dt} = -i\Delta_{pu}a + iG_0 a q + E_{pu} + E_s e^{-i\delta t}, \quad (2)$$

$$\frac{d^2 q}{dt^2} + \omega_m^2 q = \omega_m G_0 a^\dagger a. \quad (3)$$

In what follows we ignore the quantum properties of a and q [27–29]. Also let $\langle a \rangle$, $\langle a^\dagger \rangle$ and $\langle q \rangle$ be the expectation values of operators a , a^\dagger , and q . Thus by adding the corresponding noise terms, the semiclassical equations for a and q will be

$$\frac{d\langle a \rangle}{dt} = -(i\Delta_{pu} + \kappa)\langle a \rangle + iG_0 \langle a \rangle \langle q \rangle + E_{pu} + E_s e^{-i\delta t}, \quad (4)$$

$$\frac{d^2 \langle q \rangle}{dt^2} + \gamma_m \frac{d\langle q \rangle}{dt} + \omega_m^2 \langle q \rangle = \omega_m G_0 \langle a^\dagger \rangle \langle a \rangle, \quad (5)$$

where γ_m is the damping rate of mechanical mode. In order to solve these equations, we make the ansatz [30]: $\langle a(t) \rangle = a_0 + a_+ e^{-i\delta t} + a_- e^{i\delta t}$ and $\langle q(t) \rangle = q_0 + q_+ e^{-i\delta t} + q_- e^{i\delta t}$. Upon substituting these equations into equations (4) and (5), and upon working to the lowest order in E_s but to all orders in E_{pu} , we obtain in the steady state

$$a_+ = E_s \left[\frac{-i\delta + (-i\Delta_{pu} + \kappa) + C}{(\kappa - i\delta)^2 + (\Delta_{pu} + iC)^2 - D} \right], \quad (6)$$

where $A = \frac{G_0^2}{\omega_m^2}$, $B = \frac{\omega_m^2}{\omega_m^2 - i\gamma_m \delta - \delta^2}$, $C = iA\omega_m w_0 + iAB\omega_m w_0$, $D = A^2 B^2 \omega_m^2 w_0^2$ and $w_0 = |a_0|^2$. Here parameter w_0 is determined by the equation:

$$w_0 [\kappa^2 + (\Delta_{pu} - \frac{G_0^2}{\omega_m} w_0)^2] = E_{pu}^2. \quad (7)$$

In order to investigate the dispersion and absorption property of the system, we need to calculate the output field by using the input-output relation $a_{out}(t) + a_{in}(t) = \sqrt{2\kappa}a(t)$ [31], where $a_{out}(t)$ is the output operator and $a_{in}(t)$ is the input operator with zero mean value. In accordance with the above discussions, we also ignore the quantum properties of $a_{out}(t)$ and $a_{in}(t)$, and thus we can obtain

$$\langle a_{out}(t) \rangle = a_{out0} + a_{out+} e^{-i\delta t} + a_{out-} e^{i\delta t} = \sqrt{2\kappa} (a_0 + a_+ e^{-i\delta t} + a_- e^{i\delta t}). \quad (8)$$

From this equation, we see that a_{out+} equals to $\sqrt{2\kappa}a_+$, which is a parameter in analogy to the linear optical susceptibility. The real part of a_{out+} exhibits absorptive behavior, and its imaginary part shows dispersive property, for the reason that the phase of light changes $\frac{\pi}{2}$ on the reflection.

In such a system, the transmission of the probe beam is given by

$$t_p(\omega_s) = \frac{a_{out}}{a_{in}} = 1 - a_{out+} = \frac{P^2 - Q^2 - E_s \sqrt{2\kappa} (MP - NQ)}{P^2 - Q^2} + i \frac{\sqrt{2\kappa} (NP + MQ)}{P^2 - Q^2}, \quad (9)$$

where $M = \kappa - G\gamma_m \delta E$, $N = G(F+1) - \delta - \Delta_{pu}$, $P = \kappa^2 - \delta^2 + \Delta_{pu}^2 + G^2(2F+1) - 2\Delta_{pu}G(F+1)$, $Q = -2\kappa\delta + 2G^2\gamma_m^2 \delta E - 2\Delta_{pu}G\gamma_m \delta E$, $E = \frac{\omega_m^2}{(\omega_m^2 - \delta^2)^2 + \gamma_m^2 \delta^2}$, $F = E(\omega_m^2 - \delta^2)$ and $G = A\omega_m w_0$. Then the phase of signal light can be written as follows

$$\phi(\omega_s) = \arg(t_p(\omega_s)) = \arg\left(\frac{E_s \sqrt{2\kappa} (NP + MQ)}{P^2 - Q^2 - E_s \sqrt{2\kappa} (MP - NQ)}\right). \quad (10)$$

This rapid phase dispersion can lead to a group delay τ_g given by

$$\tau_g = -\frac{d\phi}{d\omega_s} = E_s \sqrt{2\kappa} \left(\frac{(NP + MQ)[2PP' - 2QQ' - E_s \sqrt{2\kappa} (M'P + MP' - N'Q - NQ')]}{[P^2 - Q^2 - E_s \sqrt{2\kappa} (MP - NQ)]^2 + 2E_s^2 \kappa (NP + MQ)^2} \right)$$

$$- \frac{(N'P + NP' + M'Q + MQ')[P^2 - Q^2 - E_s\sqrt{2\kappa}(MP - NQ)]}{[P^2 - Q^2 - E_s\sqrt{2\kappa}(MP - NQ)]^2 + 2E_s^2\kappa(NP + MQ)^2}, \quad (11)$$

where $M' = -G\gamma_m(E + \delta E')$, $N' = GF' - 1$, $P' = -2\delta + 2G^2F' - 2\Delta_{pu}GF'$, $Q' = -2\kappa + 2G^2\gamma_m^2(E + \delta E') - 2\Delta_{pu}G\gamma_m(E + \delta E')$, $E' = -\frac{E^2}{\omega_m^2}[4(\omega_m^2 - \delta^2)\delta + 2\gamma_m^2\delta]$ and $F' = E'\omega_m^2 - (E'\delta^2 + 2\delta E)$. Obviously, this analytic expression for the induced time delay is very complicated, so we have to calculate it numerically (see Fig.3 below).

III. RESULTS AND DISCUSSIONS

Before proceeding, we note that such an optomechanical system contains an optical property which is in analogous to the electrostriction or optical absorption in real material systems. As shown in Fig.1(a), when the pump field turns on, the circulating light illuminates on and gives rise to a radiation pressure force that deflects the mirror. In turn, the change of cavity's length alters the distribution of circulating intensity. This variation acts as an all optical diffraction grating in the cavity field moving back and forth with the oscillation frequency ω_m of the mechanical resonator. While the signal light travels in the cavity, the mutual interaction between input lights and the grating leads to the scattering of photons. If the signal light moves in the same direction as the diffraction grating, pump photons will be scattered into signal light, and hence a Stokes process occurs. On the contrary, if they move in different directions, signal light will be scattered into pump field, which results in an anti Stokes process. Such a behavior is very similar to the stimulated Brillouin scattering (SBS) in real material systems, in which an acoustic wave of frequency Ω is produced by the mutual interaction between light fields and material system. Through the process of electrostriction, the material system responses to the input fields by the fluctuations of dielectric constant which act as a moving diffraction grating with frequency Ω as shown in Fig.1(b) [1, 30]. In view of Kramers-Kronig relations, the cavity optomechanical system will display a strong dispersive property while gain or loss resonance occurs, and therefore the control of light group velocity can be achieved.

Then, to prove this basic idea, we choose a realistic optomechanical system [32] to illustrate the numerical results. Fig.2 plots both the real part and the imaginary part of a_{out+} as a function of signal-cavity detuning with $\Delta_{pu} = \mp 10 MHz$ respectively. Other parameters used in calculation are $E_{pu} = 2 MHz$, $\kappa = 2\pi \times 215 KHz$, $\omega_m = 10 MHz$ and

$\gamma_m = 2\pi \times 140Hz$ [32]. It is clear that the dispersion curves (Fig.2(b) and Fig.2(d)) are very steep around the center, which leads to the variation of light group velocity. At the same time the absorption spectrum (Fig.2(a) and Fig.2(c)) splits into two peaks (Normal Mode Splitting) at $\Delta_s = 0$ which ensures that the signal light passes through with little energy loss. In Fig.3(a) and Fig.3(b) we plot τ_g as a function of the amplitude of pump field E_{pu} with $\Delta_{pu} = \mp 10MHz$ respectively. While cavity-pump detuning is fixed at $-10MHz$, the slope of the dispersion curve is positive and a slow-light process occurs. As we can see, the group velocity of signal light is very sensitive to E_{pu} . In Fig.3(a), the slope becomes steeper as E_{pu} decreases, leading to an increasingly lower group velocity. Similarly, if the cavity-pump detuning shifts from $-10MHz$ to $10MHz$, the signal pulse will experience a fast-light process. Therefore it is possible to realize the switch between slow and superluminal light by simply choosing a proper pump-cavity detuning. The physical origin of these results is due to the interaction between signal light and cavity field, as discussed above. The fluctuation of light intensity inside cavity acts as a moving diffraction grating composed of large amount of photons, and gives rise to a large contribution to the phase dispersion. When the signal light passes through, the scattering of photons occurs which changes the transmitting time of the signal light.

We also use another model to discuss the property of the optomechanical system. Although the cavity is empty, we regard it as a material system composed of photons. As usual, we determine the group velocity of light as $v_g = \frac{c}{n + \omega_s(dn/d\omega_s)}$ [33, 34], where the refractive index $n \approx 1 + 2\pi\chi_{eff}$, and then $\frac{c}{v_g} = 1 + 2\pi Re[\chi_{eff}(\omega_s)]_{\omega_s=\omega_c} + 2\pi\omega_s Re(\frac{d\chi_{eff}}{d\omega_s})_{\omega_s=\omega_c}$. Here χ_{eff} is the effective susceptibility and is in direct proportion to a_{out+} . Noticing that the phase of light has changed on the reflection and $Im[\chi_{eff}(\omega_s)]_{\omega_s=\omega_c} = 0$, the group velocity index should be written as $n_g = \frac{c}{v_g} \approx 2\pi\omega_c Im(\frac{da_{out+}}{d\omega_s})_{\omega_s=\omega_c} \propto Im(\frac{da_{out+}}{d\omega_s})_{\omega_s=\omega_c}$. In Fig.3(c) and Fig.3(d), we plot the group velocity index as a function of the amplitude of pump field. The results are similar to that of Fig.3(a) and Fig.3(b).

IV. CONCLUSIONS

In conclusion, we have presented the slow light and the superluminal light in a cavity optomechanical system which is composed of an optical cavity and a mechanical resonator by a fully all-optical method. The control of light group velocity is achieved with no specific

materials placed inside the cavity. Also the switching between slow and fast light can be realized easily by simply adjusting the cavity-pump detuning. Finally, we hope that our scheme proposed here can be realized by experiment in the near future.

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Figure Captions

Fig.1 (a) Schematic diagram of a Fabry-Perot cavity with a movable mirror in the presence of a strong pump field and a weak signal field. (b) Schematic diagram of the SBS process in a real material system. ω_{pu} and ω_s are the frequencies of pump field and signal field respectively, and Ω is the frequency of acoustic wave in the material system.

Fig.2 Plot of both the real part and the imaginary part of a_{out+} with $\Delta_{pu} = \mp 10MHz$ respectively. Other parameter values are $E_{pu} = 2MHz$, $\omega_m = 10MHz$, $\kappa = 2\pi \times 215KHz$, $\gamma_m = 2\pi \times 140Hz$ and $G_0 = 1.2MHz$.

Fig.3 (a) and (b) are group delay $\tau_g = -\frac{d\phi}{d\omega_s}$ of signal light as a function of the amplitude of driving field with $\Delta_{pu} = \mp 10MHz$ respectively. (c) and (d) are the group velocity index $n_g = \frac{c}{v_g}$ of slow light versus E_{pu} with $\Delta_{pu} = \mp 10MHz$ respectively. Other parameter values are $\omega_m = 10MHz$, $\kappa = 2\pi \times 215KHz$, $\gamma_m = 2\pi \times 140Hz$ and $G_0 = 1.2MHz$.

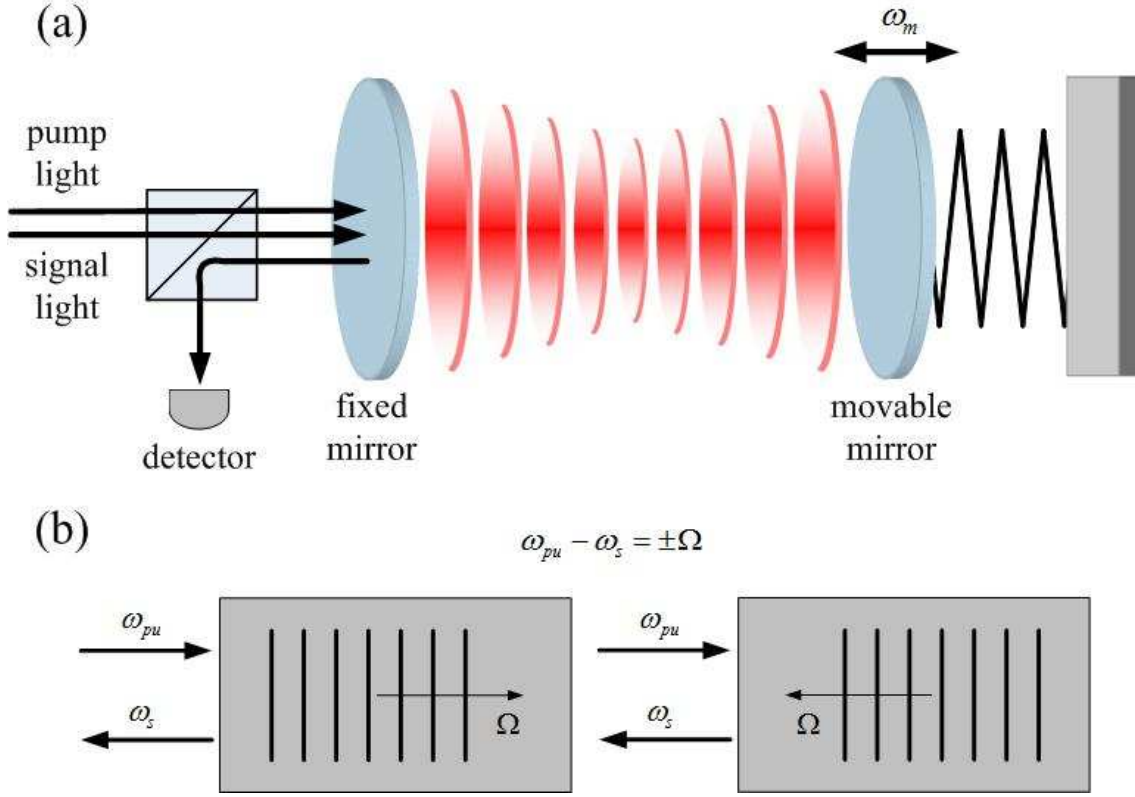


FIG. 1: (a) Schematic diagram of a Fabry-Perot cavity with a movable mirror in the presence of a strong pump field and a weak signal field. (b) Schematic diagram of the SBS process in a real material system. ω_{pu} and ω_s are the frequencies of pump field and signal field respectively, and Ω is the frequency of acoustic wave in the material system.

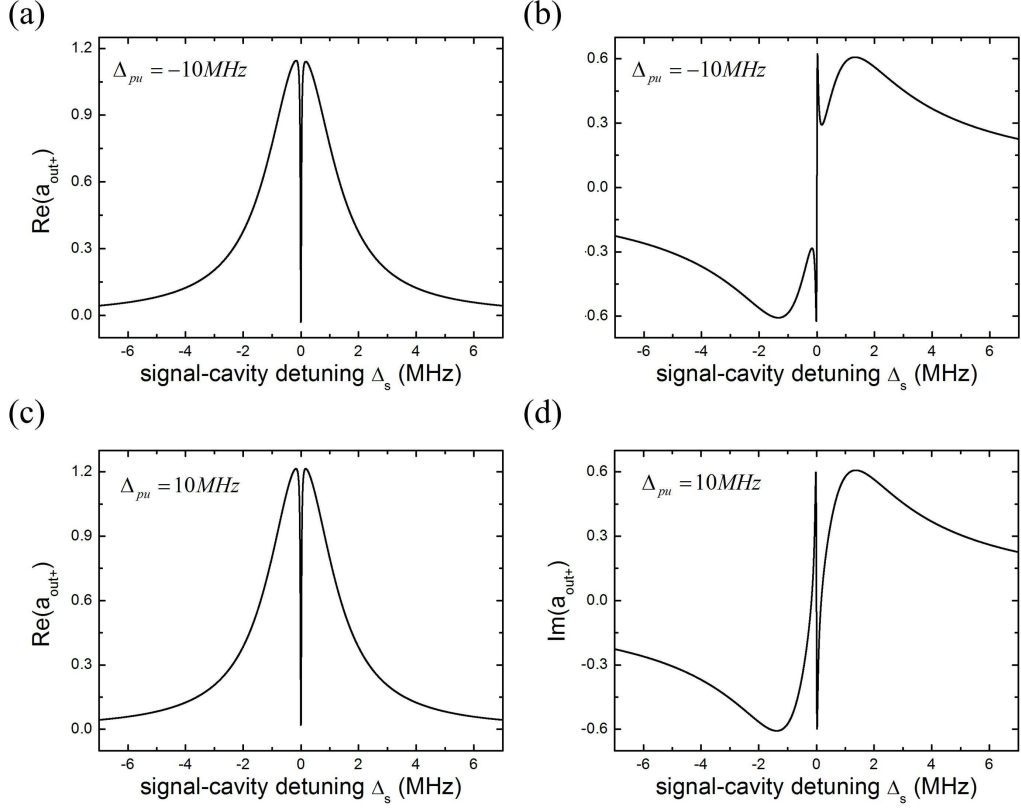


FIG. 2: Plot of both the real part and the imaginary part of a_{out+} with $\Delta_{pu} = \mp 10\text{MHz}$ respectively. Other parameter values are $E_{pu} = 2\text{MHz}$, $\omega_m = 10\text{MHz}$, $\kappa = 2\pi \times 215\text{KHz}$, $\gamma_m = 2\pi \times 140\text{Hz}$ and $G_0 = 1.2\text{MHz}$.

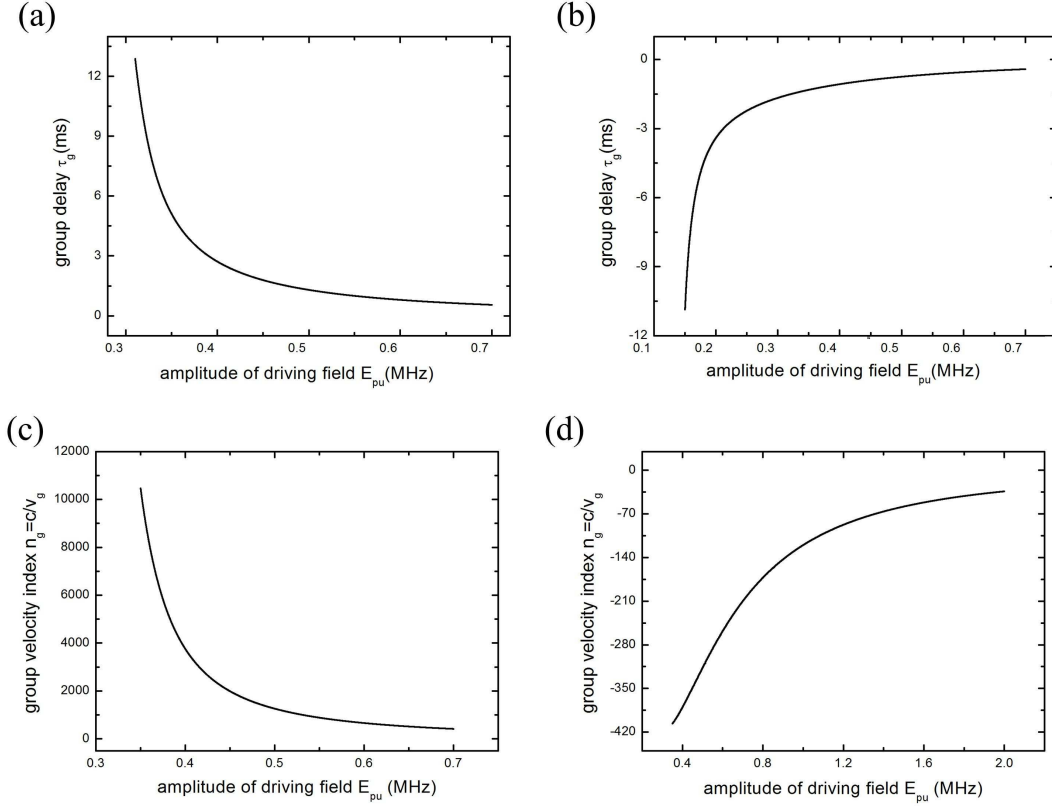


FIG. 3: (a) and (b) are group delay $\tau_g = -\frac{d\phi}{d\omega_s}$ of signal light as a function of the amplitude of driving field with $\Delta_{pu} = \mp 10 \text{ MHz}$ respectively. (c) and (d) are the group velocity index $n_g = \frac{c}{v_g}$ of slow light versus E_{pu} with $\Delta_{pu} = \mp 10 \text{ MHz}$ respectively. Other parameter values are $\omega_m = 10 \text{ MHz}$, $\kappa = 2\pi \times 215 \text{ KHz}$, $\gamma_m = 2\pi \times 140 \text{ Hz}$ and $G_0 = 1.2 \text{ MHz}$